Meriam 6/6 : Work & Energy – 2D Bodies

Recall for particles:

(i) 'Snapshot' problems: Find $\vec{a}$ at an instant in time:

$$\rightarrow \quad \Sigma \vec{F} = m\vec{a}$$

(ii) 'Journey' problems: Find $\vec{v}$ or position of a particle after a trip taken over a period of time:

$$\rightarrow \quad \vec{v}_2 = \vec{v}_1 + \int_{t_1}^{t_2} \vec{a}\, dt$$

$$\text{or} \quad \dot{\theta}_2 = \dot{\theta}_1 + \int_{t_1}^{t_2} \ddot{\theta}\, dt$$

$$T_2 = T_1 + U_{1\rightarrow 2}$$

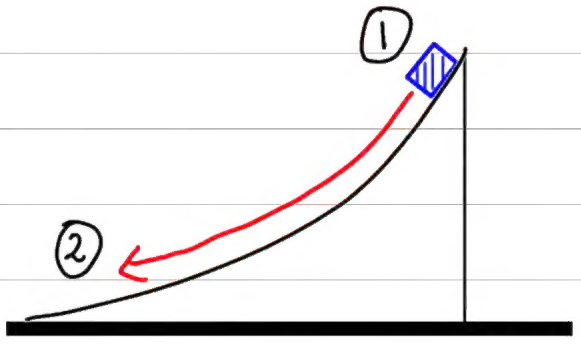
$$\vec{G}_2 = \vec{G}_1 + \int_1^2 \vec{F}\, dt$$

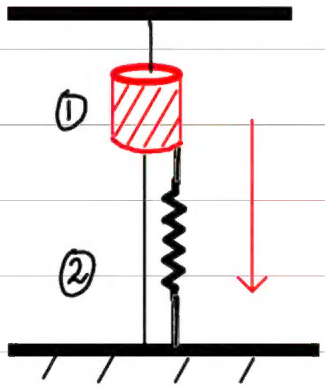
$$(\vec{H}_O)_2 = (\vec{H}_O)_1 + \int_1^2 \vec{M}_O\, dt$$

$$(\vec{H}_G)_2 = (\vec{H}_G)_1 + \int_1^2 \vec{M}_G\, dt$$

Find v_2

work-energy solution technique preferred





We can extend these same concepts to 2D bodies

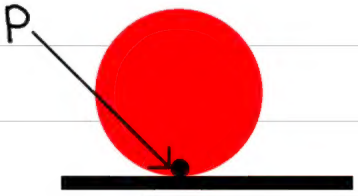
Snapshot Equations

1/ $\Sigma \vec{F} = m \vec{a}_G$

2/ $\Sigma \vec{M}_O = I_O \vec{\alpha}$ +↻

3/ $\Sigma \vec{M}_G = I_G \vec{\alpha}$ +↻

4/ $\Sigma \vec{M}_P = I_P \vec{\alpha}$ +↻ for rolling wheel



Journey Equations

Work/Energy Techniques – Meriam 6/6

Momentum Techniques – Meriam 6/8

Consider $\boxed{T_2 = T_1 + U_{1\to 2}}$ for a 2-D body:

Spring work: $\boxed{U_e = -\frac{1}{2} k (x_2^2 - x_1^2)}$

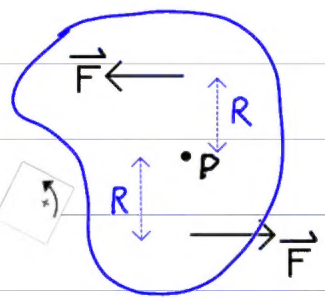
gravity work:

$U_g = -mg\Delta h$ (particle)

$U_g = -\Sigma m_i g \Delta h_i$ (many particles)

$\boxed{U_g = -mg\Delta h_G}$ (solid body)

U_{couple}: $M_P = FR + FR = 2FR$



Rotate by an angle $d\theta$

→ Forces $\vec{F}$ act through a displacement $R\,d\theta$

→ Work done $= 2((F)(R\,d\theta)) = M_P\,d\theta$

→ $\boxed{U_{couple} = \int M\,d\theta}$, → For constant M, $\boxed{U_{couple} = M\Delta\theta}$

kinetic energy T of a 2-D body:

Two possibilities:

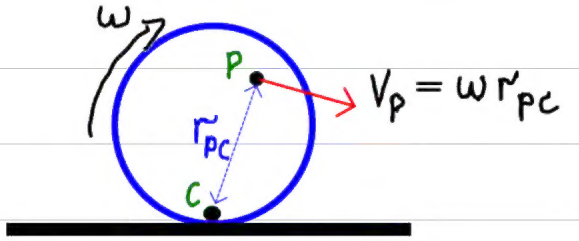
(a) $\omega = 0$ → translation without rotation

→ All points on the body travel at $\vec{V}_G$

→ $\boxed{T = \frac{1}{2} m V_G^2 \quad \text{if} \quad \omega = 0}$

(b) $\omega \neq 0$: → There exists an instantaneous center of zero velocity "C" (i.c)

→ For an instant, every point "P" on the body is circling "C" with a speed of ωr_{PC}



Divide up the body into many small pieces, $i = 1, 2, \cdots N$

$$T = \sum \frac{1}{2} m_i V_i^2 = \sum \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} \omega^2 \sum m_i r_i^2$$

→ $T = \frac{1}{2} I_C \omega^2$, where $I_C = \boxed{I_G + m d_{GC}^2}$, C = i.c.

note: $T = \frac{1}{2} (I_G + m d_{GC}^2) \omega^2$

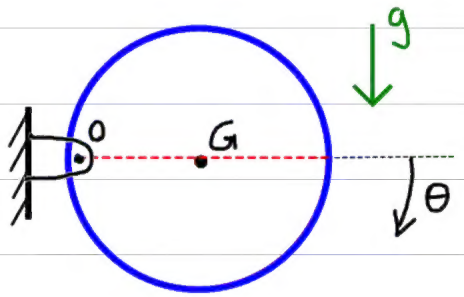
$= \frac{1}{2} m (\omega d_{GC})^2 + \frac{1}{2} I_G \omega^2$

but $V_G = |\omega d_{GC}|$

translational T rotational T

→ $\boxed{T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2}$

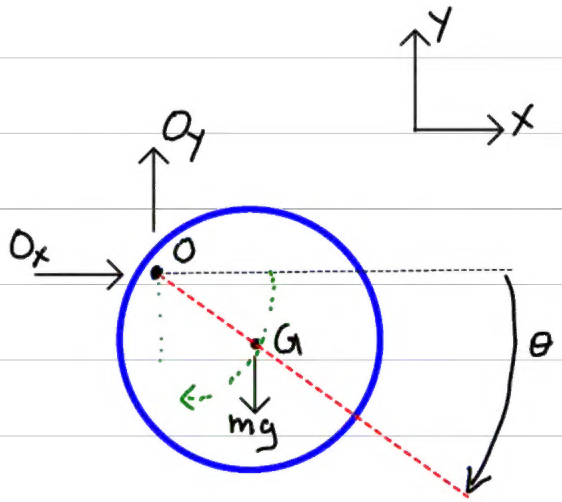
Eg.



Given: Uniform disk of radius R
mass m
Initially at $\theta = 0$ (position shown)
Released at $t = 0$, & swings down

Find: (a) $\alpha(\theta)$
(b) V_G when $\theta = 70°$

Answer:



(a) 'Snapshot' problem: FBD

$$\Sigma M_O = I_O \alpha$$

$$(mg)(R\cos\theta) = (I_G + mR^2)\alpha$$

$I_G = mR^2/2$

$$\rightarrow \quad mgR\cos\theta = 3/2\ mR^2\alpha$$

$$\rightarrow \quad \boxed{\alpha = 2/3\ \frac{g\cos\theta}{R}}$$

$$\& \quad \boxed{(a_G)_t = R\alpha = \frac{2g\cos\theta}{3}}$$

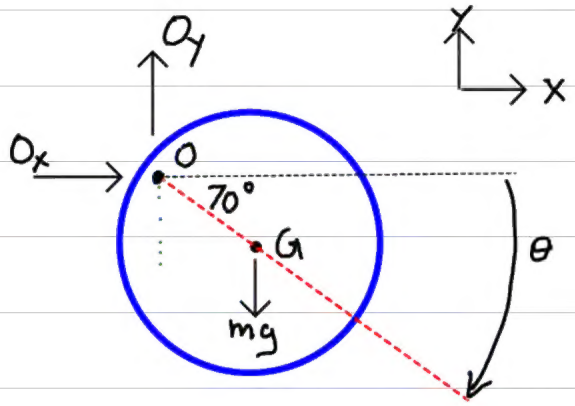
(b) "Journey" problem: circular motion around "O"

$$V_G(\theta) = \int_0^t (a_G)_t\, dt$$

Difficult

Try energy methods:

Find V_G when θ reaches 70°



$$T_2 = \cancel{T_1}^{\,0} + U_{1\to 2}$$

$$\tfrac{1}{2} I_O \omega_2^2 = -mg\Delta h$$

$$\tfrac{1}{2}\left(\tfrac{1}{2}\cancel{m}R^2 + \cancel{m}R^2\right)\omega_2^2 = -\cancel{m}g\left(-\cancel{R}\sin\theta_2\right)$$

$$\rightarrow \omega_2 = \sqrt{4g\sin\theta_2/3R} \quad \Big| \; \theta_2 = 70°$$

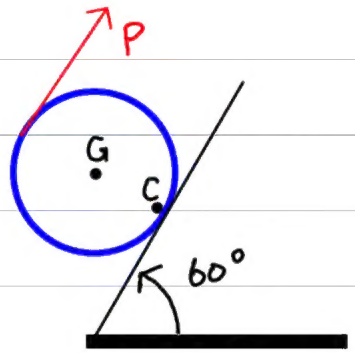
$$\vec{V}_G = \omega_2 R\,\hat{e}_t = \sqrt{4gR\sin 70°/3}\,\left(-\sin 70°\,\hat{i} - \cos 70°\,\hat{j}\right)$$

E.g. Given: R

I_G

$V_0 = 0$

No slip

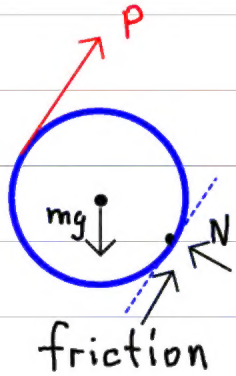


Find $\vec{V}_G$ when G has moved 5m uphill.

$$T_2 = \cancel{T_1}^{\,0} + U_{1\to 2}$$

$$\tfrac{1}{2}\cancel{I_C}^{\,(I_G+mR^2)}\omega_2^2 = -mg\cancel{\Delta h}^{\,5\sin 60} + (P)(\text{distance})^{\,(2)(5\text{ meters})}$$

$$\rightarrow \omega_2^2 = \frac{20P - 10mg\sin 60°}{I_G + mR^2}$$



$$\rightarrow \vec{V}_G = \omega_2 R\,\hat{i} = \sqrt{\frac{20PR^2 - 10mgR^2\sin 60}{I_G + mR^2}}\;\hat{i}\ \text{m/s}$$

28(f)

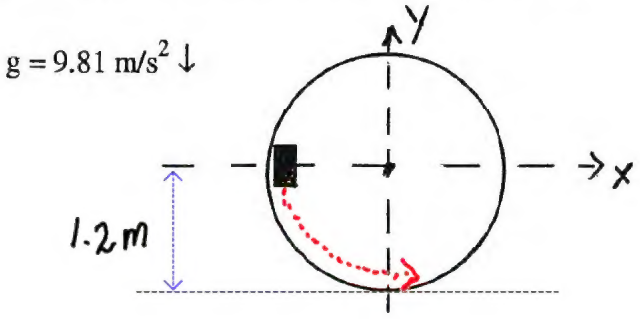
LAST Name: ANSWER

First Name: _____

Student Number: _____

MIE 200F - Quiz number 7b - November 22/00
quiz duration = 25 minutes

A 5-kg block of metal is released from rest at the position shown and is allowed to slide down the inside wall of a pipe. The inner radius of the pipe is 1.2 meters, and has a frictionless surface. What will be the maximum speed attained by the block? The radius of gyration of the block about its center of gravity is 0.3 meters. (Caution: note that the block will be translating and rotating, due to the curvature of the pipe wall.)



$T_2 = T_1 + U_{1\to 2}$

$I_{c\ of\ g}$ for a rod $= m L^2/12$ $\quad I_o = I_{c\ of\ g} + m d^2$

$I_{c\ of\ g}$ for a uniform disk $= m R^2/2$ $\quad I_{c\ of\ g} = mk^2$

$\vec{V}_P = \vec{V}_Q + \vec{V}_{P/Q}$ $\quad |V_{P/Q}| = \omega R_{PQ}$

$\vec{a}_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha r)\,\hat{e}_\theta$ $\quad \vec{a}_P = \vec{a}_Q + \vec{a}_{P/Q}$

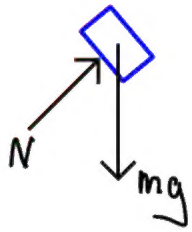
$T = \frac{1}{2} m (v_{c\ of\ g})^2 + \frac{1}{2} I_{c\ of\ g}\, \omega^2 = \frac{1}{2} I_o\, \omega^2$

$T_2 = T_1 + U_{1\to 2}$

$\frac{1}{2} m v_2^2 + \frac{1}{2} I_{cfg}\, \omega^2 = 0 - mgah$ ..

an approximation

$\frac{1}{2} m v_2^2 + \left(\frac{1}{2}\right)(mk^2)\left(\frac{v_2}{1.2}\right)^2 = -(m)(9.81)(-1.2)$



$v_2^2\left(\frac{1}{2} + \frac{1}{2}\frac{0.3^2}{1.2^2}\right) = +(9.81)(1.2)$

$v_2^2\,(0.53125) = 11.77$

$v_2 = 4.707$ m/s.

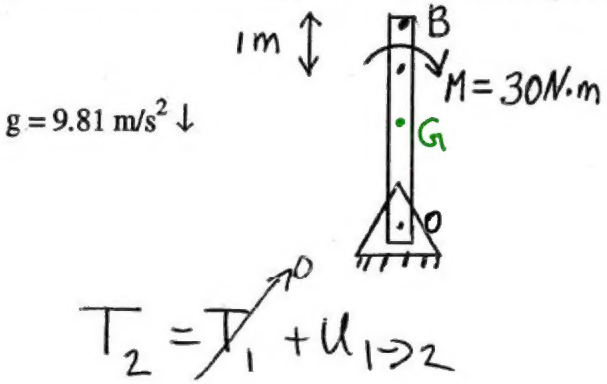
LAST Name: _____

First Name: _____

Student Number: _____

MIE 200F - Quiz number 7a - November 22/99
quiz duration = 25 minutes

A thin rod has a length of 5 meters and mass of 2 kg. It is pinned at one end, and is initially held stationary in the position shown in the diagram. A constant moment equal to 30 N·m is applied at the position shown. If the rod is released at the position shown in the diagram, what will be the speed of "B" when the rod reached a horizontal position?



$T_2 = T_1 + U_{1\to 2}$

$I_{c\ of\ g}$ for a rod = $m L^2/12$ $\quad I_o = I_{c\ of\ g} + m d^2$

$I_{c\ of\ g}$ for a uniform disk = $m R^2/2$

$V_P = V_Q + V_{P/Q}$ $\quad |V_{P/Q}| = \omega R_{PQ}$

$a_{P/Q} = -(\omega^2 R_{PQ})\hat{e}_r + (\alpha R_{PQ})\hat{e}_\theta$ $\quad a_P = a_Q + a_{P/Q}$

$U_{gravity} = -mg\Delta h$ $\quad U_{moment} = M\Delta\theta$

$T_2 = T_1 + U_{1\to 2}$

$\frac{1}{2} I_o \omega_2^2 = -mg\Delta h_G + M\Delta\theta$

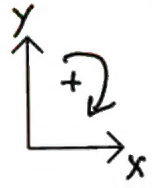
$$I_o = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2 = mL^2/3 = (2)(5)(5)/3 = 16.7$$

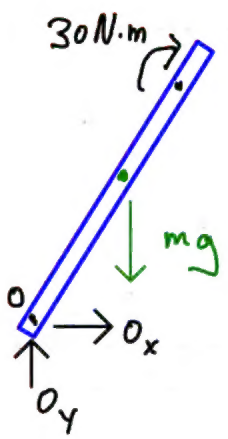
$$\left(\tfrac{1}{2}\right)(16.7)\,\omega_2^2 = -(2)(9.81)(-2.5) + (30)(\pi/2)$$

$$8.33\,\omega_2^2 = 96.17 \Rightarrow \omega_2 = 3.4\,s^{-1}\ \circlearrowleft$$

$$\Rightarrow |\vec{V}_B| = (\omega_2)(L) = (3.4)(5) = 17\ m/s$$

Note: The location at which the moment M is applied does not matter.





Summary

- Energy/work Equation for solid bodies:

$$T_2 = T_1 + U_{1\to 2}$$

- For a solid body

$$T = 1/2\, m\, v_G^2 + \tfrac{1}{2} I_G \omega^2$$

or $T = \tfrac{1}{2} I_C \omega^2$, C = instaneous center of zero velocity, i.c.

Next Lecture

- Review session - sample problems on 2D kinetics